**Western Australian Certificate of Education
Examination, 2012**

Question/Answer Booklet

MATHEMATICS: SPECIALIST 3C/3D

**Section One:
Calculator-free**

Please place your student identification label in this box

Student Number: In figures   ☐☐ ☐☐☐ ☐☐

             In words   _____

                             _____

Number of additional
answer booklets used
(if applicable):

Time allowed for this section
Reading time before commencing work: five minutes
Working time for this section: fifty minutes

Materials required/recommended for this section
To be provided by the supervisor
This Question/Answer Booklet
Formula Sheet

To be provided by the candidate
Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction tape/fluid, eraser, ruler, highlighters

Special items: nil

Important note to candidates
No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of total exam
Section One: Calculator-free	7	7	50	50	33⅓
Section Two: Calculator-assumed	12	12	100	100	66⅔
			Total	150	100

Instructions to candidates

1. The rules for the conduct of Western Australian external examinations are detailed in the *Year 12 Information Handbook 2012*. Sitting this examination implies that you agree to abide by these rules.

2. Answer the questions according to the following instructions.

    Section One: Write answers in this Question/Answer Booklet. Answer all questions.

    Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat an answer to any question, ensure that you cancel the answer you do not wish to have marked

    It is recommended that you **do not use pencil**, except in diagrams.

3. You must be careful to confine your responses to the specific questions asked and to follow any instructions that are specific to a particular question.

4. Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.
    *   Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
    *   Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question that you are continuing to answer at the top of the page.

5. The Formula Sheet is **not** handed in with your Question/Answer Booklet.

DO NOT WRITE IN THIS AREA AS IT WILL BE CUT OFF

Section One: Calculator-free   **(50 Marks)**

This section has **seven (7)** questions. Answer **all** questions. Write your answers in the spaces provided.

Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.
- Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
- Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question(s) that you are continuing to answer at the top of the page.

Working time: 50 minutes.

Question 1   **(6 marks)**

(a)   Evaluate $\displaystyle\int_{-\pi}^{\pi} |\sin x|\, dx$.   (3 marks)

(b)   Determine $\displaystyle\int 6x^2 e^{x^3+9}\, dx$.   (3 marks)

Question 2   **(8 marks)**

Let the 2×2 matrix $A = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$.

(a)   Evaluate A^2 and A^3.   (2 marks)

(b)   Given your answers to (a), conjecture the form of the matrix A^n where n is a positive
        integer. Use proof by mathematical induction to establish the truth of your conjecture.
                                                                      (4 marks)

(c)   Determine the effect of applying the matrix A^2 to the rectangle with vertices (1,0), (3,0), (3,1) and (1,1). What type of transformation does the matrix A^2 represent?   (2 marks)

See next page

Question 3   **(7 marks)**

(a)   Evaluate $\displaystyle\int_0^{\frac{\pi}{3}} \frac{6\sin 3x - 8\cos 2x}{\cos 3x + 2\sin 2x}\,dx$.   (3 marks)

(b)   Use the substitution $u = \ln x$ to determine $\displaystyle\int \frac{\sqrt{\ln x} + \ln\sqrt{x}}{x}\,dx$.   (4 marks)

Question 4   **(6 marks)**

In the triangle ABC the sides BC and AC have midpoints M and N respectively. Lines perpendicular to these sides are drawn through their respective midpoints and intersect at the point O.

Denote the line segments $\overrightarrow{OA}$ = **a**, $\overrightarrow{OB}$ = **b** and $\overrightarrow{OC}$ = **c**.
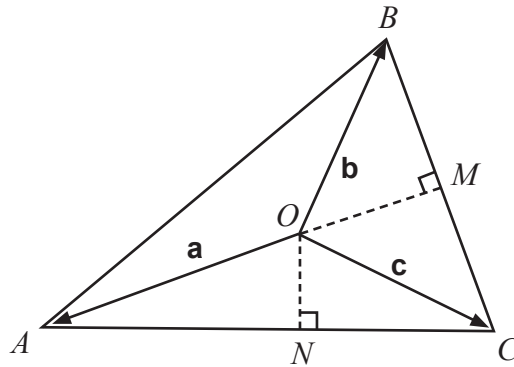
(a)   Express $\overrightarrow{ON}$ in terms of **a** and **c** and $\overrightarrow{OM}$ in terms of **b** and **c**.   (1 mark)

(b)   Show that $|\mathbf{a}|=|\mathbf{b}|=|\mathbf{c}|$.   (2 marks)

(c)   If P is the midpoint of side AB, show that $\overrightarrow{OP}$ is perpendicular to $\overrightarrow{AB}$.   (3 marks)

Question 5   **(11 marks)**

A mining drill moves through a cross-section of rock such that its coordinates x and y satisfy:

$$\frac{dx}{dt} = -2\sin t, \quad \frac{dy}{dt} = -6\sin 2t$$

where x and y are measured in metres and t is the time expressed in minutes.
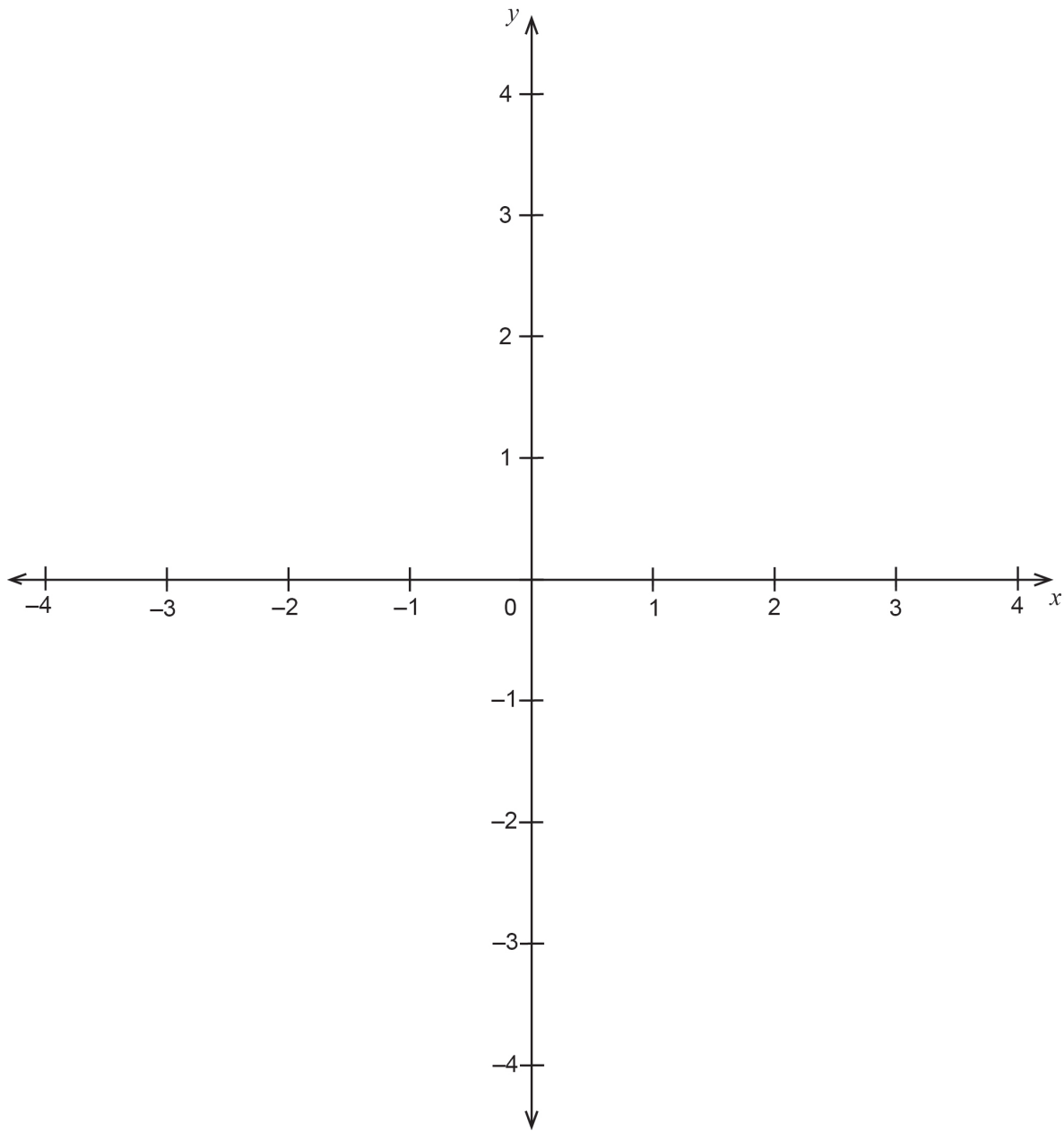Initially $x = 2$ and $y = 3$.

(a)   Determine $\dfrac{dy}{dx}$ when $t = \dfrac{\pi}{6}$.   (2 marks)

(b)   Determine the position of the drill when $t = \pi$.   (3 marks)

(c)   Determine the Cartesian equation of the path of the drill.   (3 marks)

(d)   Sketch the path of the drill. Label clearly the points on the path corresponding to the two
        times $t = 0$ and $t = \pi$. (3 marks)

Question 6   **(7 marks)**

Describe geometrically the set of points $\mathbf{r} = (x, y, z)$ that satisfy each of the following vector equations in which $\mathbf{a}$ denotes a non-zero constant vector.

(a) $|\mathbf{r} - \mathbf{a}| = 3$ (2 marks)

(b) $(\mathbf{r} - \mathbf{a}) \bullet \mathbf{a} = 0$ (2 marks)

(c) $(\mathbf{r} - \mathbf{a}) \bullet \mathbf{r} = 0$ (3 marks)

Question 7 **(5 marks)**

(a) A square matrix is said to be singular if it has no inverse.

Show that the matrix $X = \begin{bmatrix} \cos 72° & \sin 18° \\ \sin 72° & \cos 18° \end{bmatrix}$ is singular. (2 marks)

(b) Let A be any square matrix such that $A^3 = 0$.
Prove by contradiction that A must be singular. (3 marks)

End of questions

Additional working space

Question number: _____

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